Weakly monotone finite volume scheme for parabolic and diffusion equations in strongly anisotropic porous media

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Résumé

Parabolic equations appear frequently in the modeling of many everday life applications. For example, they form part of complex systems such as flows in porous media, fluid dynamics models, as well as mass and heat transfer problems. In terms of numerical resolution of these equations by the finite volume method, several schemes have been developed, mostly focused on preserving certain properties like positivity and maximum principale. However, the monotony is quite diffucult question.
In this work, we present an original approach allowing to preserve the weak monotonicity of a finite volume discretization in the case of highly anisotropic parabolic equations. In other words, the computed numerical solution honors the physical ranges of the initial condition. The main idea lies in devising a nonlinear damping parameter eliminating the problematic fluxes when they occur. We check that the structure of the scheme naturally ensures the weak monotonicity of the approximate solutions. We also establish energy estimates, which leads to a proof of existence of the numerical solutions.

Several numerical test cases demonstrate the proposed approach's ability to maintain the physical ranges of the solution, as well as to provide good accuracy and robustness with respect to the mesh and high ratios of anisotropy. Finally, we apply this novel methodology to simulate mass transfer in hygroscopic media, fucusing on mass diffusion inside within wood.

Références

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